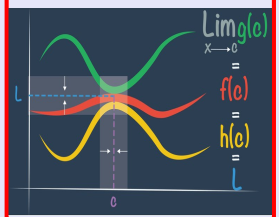
Math 261

Spring 2023

Lecture 46



Feb 19-8:47 AM

Use Subs. method to find

$$\int x^3(1+x^4)^5\,dx$$

$u = 1 + x^4$

$du = 4x^3\,dx$

$\frac{du}{4} = x^3\,dx$

$$= \int u^5 \cdot \frac{du}{4} = \frac{1}{4}\int u^5\,du = \frac{1}{4}\cdot\frac{u^6}{6} + C$$

$$\boxed{= \frac{1}{24}(1+x^4)^6 + C}$$

check:

$$\frac{d}{dx}\left[\frac{1}{24}(1+x^4)^6 + C\right] = \frac{1}{\cancel{24}}\cdot\cancel{6}(1+x^4)^5\cdot\cancel{4}x^3 + 0$$

$$= \underbrace{x^3(1+x^4)^5}_{\text{Integrand}}$$

May 9-8:49 AM

$$\int x \sec^2(x^2+1)\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$= \int \sec^2 u \cdot \frac{du}{2} = \frac{1}{2}\int \sec^2 u\,du = \frac{1}{2}\cdot \tan u + C$$

$$= \boxed{\frac{1}{2}\tan(x^2+1) + C}$$

check

$$\frac{d}{dx}\left[\frac{1}{2}\tan(x^2+1) + C\right] = \frac{1}{\cancel{2}}\cdot \sec^2(x^2+1)\cdot \cancel{2}x + 0$$

$$= \underbrace{x\sec^2(x^2+1)}_{\text{Integrand}}$$

May 9-8:54 AM

$$\underbrace{\int \sqrt{\cot x}\cdot \csc^2 x\,dx}_{\text{Integrand}}$$

Let $u = \cot x$

$du = -\csc^2 x\,dx$

$-du = \csc^2 x\,dx$

$$= \int \sqrt{u}\cdot -du = -\int u^{1/2}\,du = -\frac{u^{1/2+1}}{1/2+1} + C$$

$$= -\frac{u^{3/2}}{3/2} + C = -\frac{2}{3}u\sqrt{u} + C$$

$$= \boxed{-\frac{2}{3}\cot x\sqrt{\cot x} + C}$$

May 9-8:59 AM

$$\int x\sqrt{x-1}\,dx \qquad u = x-1 \rightarrow u+1 = x$$

$$du = dx$$

$$= \int (u+1)\cdot\sqrt{u}\;du = \int (u+1)u^{1/2}\,du$$

$$= \int (u^{3/2} + u^{1/2})\,du$$

$$= \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{5}\sqrt{(x-1)^5} + \frac{2}{3}\sqrt{(x-1)^3} + C$$

$$= \frac{2}{5}(x-1)^2\sqrt{x-1} + \frac{2}{3}(x-1)\sqrt{x-1} + C$$

May 9-9:04 AM

$$\int x\sqrt{x-1}\,dx \qquad \text{Let } u = \sqrt{x-1}$$

$$u^2 = x-1 \rightarrow u^2+1 = x$$

$$2u\,du = dx$$

$$= \int (u^2+1)\cdot u\cdot 2u\,du$$

$$= 2\int (u^4 + u^2)\,du = 2\left[\frac{u^5}{5} + \frac{u^3}{3}\right] + C$$

$$= \frac{2}{5}u^5 + \frac{2}{3}u^3 + C$$

$$= \frac{2}{5}\left(\sqrt{x-1}\right)^5 + \frac{2}{3}\left(\sqrt{x-1}\right)^3 + C$$

$$= \boxed{\frac{2}{5}\sqrt{(x-1)^5} + \frac{2}{3}\sqrt{(x-1)^3} + C}$$

May 9-9:08 AM

Evaluate $\int_0^1 \sqrt[3]{7x+1}\,dx = \int_1^8 \sqrt[3]{u} \cdot \frac{du}{7}$

$u = 7x + 1$

$du = 7\,dx \qquad \frac{du}{7} = dx$

$x = 0 \rightarrow u = 1$

$x = 1 \rightarrow u = 8$

$$= \frac{1}{7}\int_1^8 u^{1/3}\,du$$

$$= \frac{1}{7} \cdot \frac{u^{4/3}}{4/3}\Big]_1^8$$

$$= \frac{3}{28}\, u\sqrt[3]{u}\Big]_1^8$$

$$= \frac{3}{28}\left[8\sqrt[3]{8} - 1\sqrt[3]{1}\right]$$

$$= \frac{3}{28}\left[16 - 1\right]$$

$$= \boxed{\frac{45}{28}}$$

May 9-9:12 AM

$$\int_0^{\pi/2} \underbrace{\cos x \cdot \sin(\sin x)}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$x = 0 \rightarrow u = \sin 0 = 0$

$x = \frac{\pi}{2} \rightarrow u = \sin\frac{\pi}{2} = 1$

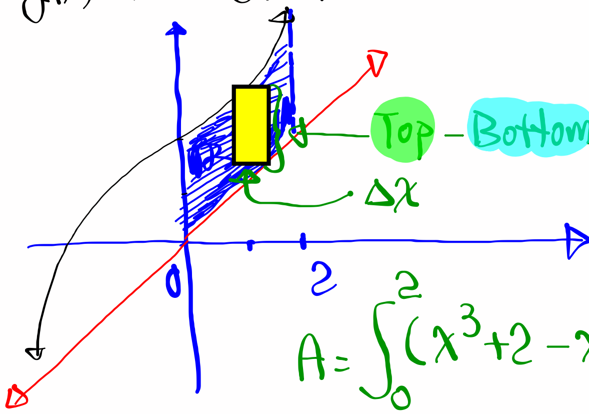
$$= \int_0^1 \sin u\,du = -\cos u\Big]_0^1 = -\left[\cos 1 - \cancel{\cos 0}^{\,1}\right]$$

$$= -1\left[\cos 1 - 1\right]$$

$$= \boxed{1 - \cos 1}$$

May 9-9:18 AM

Find the area below $f(x) = x^3 + 2$, above $g(x) = x$ from $x = 0$ to $x = 2$.



Top − Bottom $= x^3 + 2 - x$

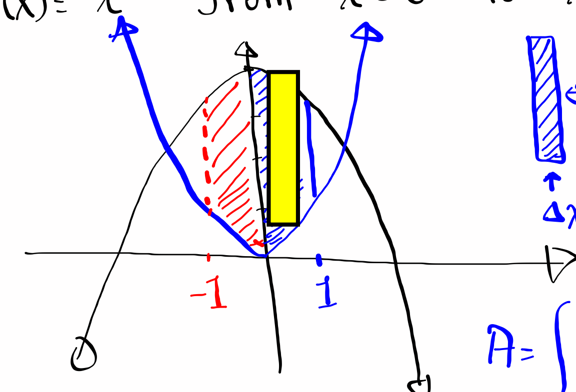
$$A = \int_0^2 (x^3 + 2 - x)\,dx = \left(\frac{x^4}{4} + 2x - \frac{x^2}{2}\right)\Bigg]_0^2$$

$$= \left(\frac{2^4}{4} + 2(2) - \frac{2^2}{2}\right) - \left(\frac{0^4}{4} + 2(0) - \frac{0^2}{2}\right)$$

$$= 4 + 4 - 2 = \boxed{6}$$

May 9-9:23 AM

Find the area below $f(x) = 4 - x^2$, above $g(x) = x^2$ from $x = 0$ to $x = 1$.



Top − Bottom $= 4 - x^2 - x^2$

$= 4 - 2x^2$

Δx

LW

$$A = \int_0^1 (4 - 2x^2) \cdot dx$$

$$= \left(4x - \frac{2x^3}{3}\right)\Bigg]_0^1 = \boxed{\frac{10}{3}}$$

If we wanted area bounded between $x = -1$ & $x = 1$

$\Rightarrow \boxed{20/3}$

May 9-9:30 AM

Evaluate

$$\int \sec^3 x \cdot \tan x \, dx$$

$$\int \sec^2 x \cdot \sec x \cdot \tan x \cdot dx$$

$u = \sec x$

$du = \sec x \cdot \tan x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int \sec^2 x \cdot \sec x \cdot \tan x \, dx$$

$$= \int u^2 du = \frac{u^3}{3} + C = \boxed{\frac{1}{3}\sec^3 x + C}$$

May 9-9:38 AM

$$\int_0^1 x^2 \sqrt{x+1} \, dx$$

$u = \sqrt{x+1}$

$u^2 = x+1 \to u^2 - 1 = x$

$2u\,du = dx$

$x = 0 \to u = 1$

$x = 1 \to u = \sqrt{2}$

$$\int_1^{\sqrt{2}} (u^2-1)^2 \cdot u \cdot 2u\,du$$

$$= 2\int_1^{\sqrt{2}} (u^4 - 2u^2 + 1) u^2 \, du$$

$$= 2\int_1^{\sqrt{2}} (u^6 - 2u^4 + u^2) \, du$$

$$= 2\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]_1^{\sqrt{2}}$$

$$= 2\left[\left(\frac{(\sqrt{2})^7}{7} - \frac{2(\sqrt{2})^5}{5} + \frac{(\sqrt{2})^3}{3}\right) - \left(\frac{1}{7} - \frac{2}{5} + \frac{1}{3}\right)\right]$$

$$= 2\left[\frac{(\sqrt{2})^6 \cdot \sqrt{2}}{7} - \frac{2(\sqrt{2})^4\sqrt{2}}{5} + \frac{(\sqrt{2})^2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}\right]$$

$$= 2\left[\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}\right]$$

$$= 2\left[\frac{15 \cdot 8\sqrt{2} - 21 \cdot 8\sqrt{2} + 35 \cdot \sqrt{2} - 15 + 21 - 35}{105}\right]$$

$$= 2\left[\frac{-13\sqrt{2} - 29}{105}\right] = \boxed{-\frac{26\sqrt{2} + 58}{105}}$$

May 9-9:45 AM